



VRC

# VALUE INSIGHTS

**VALUING SECURITIES  
IN A COMPLEX  
CAPITAL  
STRUCTURE**

# INTRODUCTION



Complex capital structures include convertible and participating preferred stock (Preferreds). These securities offer both fixed payments (FPs) and equity option payoffs. Appraisers often use the Option Pricing Model (OPM) to value these securities. The OPM is an option-theoretic valuation methodology grounded in financial option theory and is applied after enterprise or equity value has been determined using traditional valuation approaches, solely to allocate value among securities. To implement the OPM, appraisers typically estimate volatility empirically. The OPM results produce implied credit spreads for the FP.

Appraisers have found that the OPM tends to overvalue both stand-alone fixed-income securities and senior FPs and consequently undervalue common stock. This white paper examines the valuation performance of six variations of the OPM.

1. The first model treats volatility as an endogenous variable, not an empirically estimated input. The Appraiser adjusts volatility to achieve an implied credit spread for the most senior FP that matches market data.
2. The second model treats volatility as a non-constant endogenous variable. The Appraiser adjusts the volatilities of multiple breakpoints to match market credit spreads for each FP.
3. The third model discounts each FP at a market yield and values the conversion features in an OPM.
4. The fourth model adapts a method of convertible bond valuation proposed by Tsiveriotis and Fernandes<sup>1</sup> (TF). This model discounts FPs by the risk-free rate when a Preferred is in-the-money. When a Preferred is out-of-the-money, a market credit spread is used. This model values the conversion features in an OPM, as in Model 3.
5. The fifth model treats the value of a common stock share, not the value of Total Equity, as the underlying stochastic variable. This model discounts the FPs by their risky yields, as in Model 3. It values each conversion feature as a warrant on the common stock.
6. The sixth model differs from the fifth in using the TF approach to value the FPs.

The valuation results indicate that all six models have positive features. Models 3, 4, 5, and 6 exhibit certain limitations. Models 1 and 2, which differ from the OPM only in their treatment of volatility as an input, appear to show significant promise as practical alternatives to current practice.

<sup>1</sup> Tsiveriotis K. and C. Fernandes. Valuing Convertible Bonds with Credit Risk. *Journal of Fixed Income*, September 1998, Vol. 8, pp. 95-102. Bardhan, Bergier, Derman, Dosembet, and Kani developed a similar model when they were at Goldman Sachs & Company (the GS model). For detailed descriptions of both models see Grant, D. "Comparing Three Convertible Debt Valuation Models", *Business Valuation Review*, 36 1 (2017), 32 – 41, who reports, "the TF model figures much more prominently in the literature having been cited 295 times as compared to 28 times for the GS model."

# ANALYSIS



## THE BASE EXAMPLE

We illustrate this discussion with an example from the “Equity Securities Task Force” working draft, used to update the AICPA Accounting and Valuation Guide.<sup>2</sup> In this scenario, a firm raised \$15.0 million of Series C convertible preferred at \$5.00 per share. The firm previously issued \$10.0 million of Series B convertible preferred at \$2.00 per share, and \$5.0 million of Series A at \$1.00 per share. There are 5 million shares of common stock outstanding and no outstanding debt. Series C is senior to Series B, which is senior to Series A.

Management estimates a 5-year term to a liquidity event. The Appraiser assumes a volatility of 100% for Total Equity. The market-based spreads for Series C, B, and A are 20%, 24% and 29%, respectively. The risk-free rate is 1.5% per annum.

## TRADITIONAL OPM

The table below provides the OPM valuation results and the implied credit spreads of the FPs of Series C, B, and A: 15%, 28% and 33%.<sup>3</sup> The implied credit spread for Series C is lower than the assumed market indication of 20%. The implied credit spreads for Series A and Series B are higher. In addition, the difference between Series C and Series B credit spreads is significantly higher than the assumed market indication.

## BASIC OPM

### OPM Results

Equity value	\$67.5						
Breakpoints	\$0.0	\$15.0	\$25.0	\$30.0	\$35.0	\$45.0	\$90.0
Call values	\$67.5	\$60.5	\$57.7	\$56.6	\$55.5	\$53.7	\$47.8
Volatility	100%	100%	100%	100%	100%	100%	100%
Call spreads	\$7.0	\$2.8	\$1.1	\$1.0	\$1.8	\$5.9	\$47.8
Series C participation	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	16.7%
Series B participation	0.0%	100.0%	0.0%	0.0%	0.0%	33.3%	27.8%
Series A participation	0.0%	0.0%	100.0%	0.0%	50.0%	33.3%	27.8%
Common participation	0.0%	0.0%	0.0%	100.0%	50.0%	33.3%	27.8%
	<b>Total Value</b>	<b>Value per Share</b>	<b>Call Value per Share</b>	<b>Implied Spread of FP</b>			
Series C	\$15.0	\$5.00	\$2.65	15%			
Series B	\$18.0	\$3.60	\$3.05	28%			
Series A	\$17.3	\$3.46	\$3.23	33%			
Common	\$17.2	\$3.44	\$3.44				
Total Equity	\$67.5						

### Credit Spread Calculations

$$15\% = (\$15.0/\$7.0)^{(1/5)} - 1.015$$

$$28\% = (\$10.0/\$2.8)^{(1/5)} - 1.015$$

$$33\% = (\$5.0/\$1.1)^{(1/5)} - 1.015$$

<sup>2</sup> Chapter 6 Valuation of Equity Securities in Complex Capital Structures, Chapter 6 Valuation of Equity Securities in Complex Capital Structures, Example – Series C Up-Round (Senior) pp. 32 – 36, Working Draft.

<sup>3</sup> All dollar values in the tables, except per share values, are in millions.

## MODEL 1: A SINGLE ENDOGENOUS VOLATILITY

The OPM starts with volatility as a market input and back-solves for a Total Equity that matches the Series C transaction value. Model 1 back-solves for the volatility and Total Equity that match the value of Series C and an implied credit spread of 20%. In this example, that volatility is 114%. Total Equity increases from \$67.5 to \$71.7 million. Common equity increases from \$17.2 to \$18.7 million, from \$3.44 to \$3.73 per share.

The appraiser must judge whether a volatility of 100% and a credit spread for Series C of 15% is more reasonable than a volatility of 114% and a credit spread of 20%. In practice, an appraiser can investigate a range of pairs of volatility and credit spread values to identify the most reasonable choice. Because volatility and credit spreads for private companies can be challenging to estimate empirically, this more flexible approach to volatility may be preferable.<sup>4</sup>

### MODEL 1: A SINGLE ENDOGENOUS VOLATILITY

Equity value	\$71.7						
Breakpoints	\$0.0	\$15.0	\$25.0	\$30.0	\$35.0	\$45.0	\$90.0
Volatility	114%	114%	114%	114%	114%	114%	114%
Call values	\$71.7	\$66.0	\$63.9	\$63.0	\$62.2	\$60.7	\$56.0
Call spreads	\$5.7	\$2.1	\$0.9	\$0.8	\$1.4	\$4.7	\$56.0
Series C participation	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	16.7%
Series B participation	0.0%	100.0%	0.0%	0.0%	0.0%	33.3%	27.8%
Series A participation	0.0%	0.0%	100.0%	0.0%	50.0%	33.3%	27.8%
Common participation	0.0%	0.0%	0.0%	100.0%	50.0%	33.3%	27.8%
	<b>Total Value</b>	<b>Value per Share</b>	<b>Call Value per Share</b>	<b>Implied Spread of FP</b>			
Series C	\$15.0	\$5.00	\$3.11	20%			
Series B	\$19.3	\$3.86	\$3.43	35%			
Series A	\$18.8	\$3.75	\$3.57	40%			
Common	\$18.7	\$3.73	\$3.73				
Total Equity	\$71.7						

#### Credit Spread Calculations

$$20\% = (\$15.0/\$5.7)^{(1/5)} - 1.015$$

$$35\% = (\$10.0/\$2.1)^{(1/5)} - 1.015$$

$$40\% = (\$5.0/\$0.9)^{(1/5)} - 1.015$$

<sup>4</sup> We note that similar to the previous model the difference between class B and class C credit spreads is significantly higher than market indication.

## MODEL 2: THREE VOLATILITIES TO MATCH THREE CREDIT SPREADS

This model allows for variable volatility. It back-solves for the volatilities for the first, second, and third breakpoints to match the market yields of Series C, B, and A. It uses the volatility for the third breakpoint for the higher-level breakpoints.<sup>5</sup> This is a modification of one of the basic assumptions of the OPM approach. This modification is consistent with research that shows that implied market volatilities for common stock vary with strike prices. In this capital structure, the volatilities are 115%, 108% and 106%. The values of the FPs of Series A and B, as well as the total equity value, increase from Model 1. The per share values of Series B, A, and Common increase from \$3.86 to \$4.13, from \$3.75 to \$3.93, and from \$3.73 to \$3.86, respectively.

The main advantage of this approach is that the implied credit spreads match the market indications. In addition, the values of Series A, Series B, and Common are closer to the value of Series C. One of the challenges with this approach would be implementing it in Monte Carlo simulations, where the underlying stock price process will not follow a Geometric Brownian motion with a fixed volatility.<sup>6</sup>

### MODEL 2: THREE ENDOGENOUS VOLATILITIES

Equity value	\$74.6						
Breakpoints	\$0.0	\$15.0	\$25.0	\$30.0	\$35.0	\$45.0	\$90.0
Volatility	115%	115%	108%	106%	106%	106%	106%
Call values	\$74.6	\$68.9	\$65.7	\$64.4	\$63.4	\$61.7	\$56.0
Call spreads	\$5.7	\$3.2	\$1.3	\$1.0	\$1.7	\$5.7	\$56.0
Series C participation	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	16.67%
Series B participation	0.0%	100.0%	0.0%	0.0%	0.0%	33.3%	27.78%
Series A participation	0.0%	0.0%	100.0%	0.0%	50.0%	33.3%	27.78%
Common participation	0.0%	0.0%	0.0%	100.0%	50.0%	33.3%	27.78%
	<b>Total Value</b>	<b>Value per Share</b>	<b>Call Value per Share</b>	<b>Implied Spread of FP</b>			
Series C	\$15.0	\$5.00	\$3.11	20%			
Series B	\$20.7	\$4.13	\$3.49	24%			
Series A	\$19.6	\$3.93	\$3.66	29%			
Common	\$19.3	\$3.86	\$3.86				
Total Equity	\$74.6						

#### Credit Spread Calculations

$$20\% = (\$15.0/\$5.7)^{(1/5)} - 1.015$$

$$24\% = (\$10.0/\$3.2)^{(1/5)} - 1.015$$

$$29\% = (\$5.0/\$1.3)^{(1/5)} - 1.015$$

<sup>5</sup> A variation of this approach could consider the volatility to keep decreasing up to a breakpoint equal to the equity value. Such a variation could provide more flexibility in estimating volatility and yields.

<sup>6</sup> Note that there are well established methodologies to estimate the probability density function ("pdf") of stock price at expiration in the presence of volatility surfaces. Particularly, there is a non-arbitrage argument that the second derivative of call option price to strike price is equal to the pdf of stock price.



## MODEL 4:

### VALUE FIXED PAYMENTS WITH TF DISCOUNTING AND THE CONVERSION FEATURES IN THE OPM

The TF approach to valuing Preferreds discounts the FPs by two different rates depending on whether the instrument is in-the-money (conversion likely) or out-of-the-money (conversion unlikely). The discount rate is the risk-free rate when the Preferred converts, since the payoff behaves like equity. The discount rate includes a credit spread when the Preferred does not convert, reflecting default risk. Some think this spread should be a “downside spread”. The downside spread equates the value of the FP to the value produced by the market spread. Given a credit spread, you can imply a downside credit spread by equating the present values using conventional discounting to the value produced using TF discounting with a downside spread. Note that applying downside spreads in Model 4 produces results similar to those of Model 3. Model 4 is of interest only if it employs TF discounting with the usual credit spread. Moreover, TF wrote, “we provide a self-consistent framework for the use of market-observed credit spreads of straight bonds in the valuation of Convertible Bonds”.<sup>7</sup> Therefore, Model 4 explores the effect of using the TF approach to valuing the FPs without using downside spreads.

In this example, Series C promises a payment of \$15.0 million when equity value is less than \$90.0 million plus 3/18 of value of Total Equity ( $TE$ ) above \$90.0 million.<sup>8</sup>

$$\begin{aligned} \$15.0 &= \frac{\$15.0}{1.215^5} (1-N(d_2)) + \frac{3}{18} (TE)N(d_1) \text{ or} \\ \$15.0 &= \frac{\$15.0}{1.015^5} N(d_2) + \frac{\$15.0}{1.215^5} (1-N(d_2)) + \frac{3}{18} C(TE, \$90.0, \ln(1.015), 100\%, 5) \\ TE &= \$70.4 \end{aligned}$$

The BSM formula for a call option with terms of ( $TE, 90, \ln(1.015), 100\%, 5$ ) determines the values of  $N(d_1)$  and  $N(d_2)$ .  $C$  is the value of a call option.

$$\$15.0 = \frac{\$15.0}{1.015^5} 11.6\% + \frac{\$15.0}{1.215^5} 88.4\% + \frac{3}{18} \$50.3$$

The implied value of equity is \$70.4 million, and the per-share value of Common is \$3.60. Compared to Model 3, this model produces lower values for Series A, B, and Common.<sup>9, 10</sup>

<sup>7</sup> TF page 96.

<sup>8</sup> The Series C converts to 3 million common shares, and the total as-converted share count is 18 million, 3 million Series C plus 5 million each of Series B and A and Common.

<sup>9</sup> This model also has an internal inconsistency. The total value of the four securities is \$72.3 million, while the Total Equity value is \$70.4 million. Also, the call option values present an arbitrage opportunity, because they are not convex with respect to exercise prices.

<sup>10</sup> A variation of this approach could consider applying a discount to the volatility input consistent with what is done in practice in valuing the convertible bonds of public companies.

## MODEL 4:

### VALUE FIXED PAYMENTS WITH RISKY YIELDS AND THE CONVERSION FEATURES IN THE OPM

#### MODEL 4: FIXED PAYMENTS DISCOUNTED IN THE TF MANNER + OPM

Total Equity	\$70.4						
Breakpoints	\$0.0	\$15.0	\$25.0	\$30.0	\$35.0	\$45.0	\$90.0
Volatility	100%	100%	100%	100%	100%	100%	100%
Call values	\$70.4	\$63.3	\$60.5	\$59.3	\$58.2	\$56.4	\$50.3
Call spreads				\$1.1	\$1.9	\$6.1	\$50.3
Series C participation				0.0%	0.0%	0.0%	16.7%
Series B participation				0.0%	0.0%	33.3%	27.8%
Series A participation				0.0%	50.0%	33.3%	27.8%
Common participation				100.0%	50.0%	33.3%	27.8%
Series C upside = (16.7%)(\\$50.3) = \$8.4							
Series B upside = (33.3%)(\\$6.1) + (27.8%)(\\$50.3) = \$16.0							
Series A upside = (50.0%)(\\$1.9) + (33.3%)(\\$6.1) + (27.8%)(\\$50.3) = \$16.9							
Common = (100.0%)(\\$1.1) + ((50.0%)(\\$1.9) + (33.3%)(\\$6.1) + (27.8%)(\\$50.3) = \$18.0							

	Series C	Series B	Series A	Common	Total Equity
FP	\$15.0	\$10.0	\$5.0		
Spread	20.0%	24.0%	29.0%		
$N(d_2)$	11.6%	18.8%	22.0%		
Risk-free rate PV of FP	\$1.6	\$1.7	\$1.0		
$1 - N(d_2)$	88.4%	81.2%	78.0%		
Spread-adjusted rate PV of FP	\$5.0	\$2.6	\$1.0		
Probability-weighted PV of FP	\$6.6	\$4.4	\$2.1		
Call option value	\$8.4	\$16.0	\$16.9	\$18.0	
Total	\$15.0	\$20.3	\$19.0	\$18.0	\$72.3
Number of shares (mm)	3.0	5.0	5.0	5.0	
Value per share	\$5.00	\$4.07	\$3.80	\$3.60	

## MODEL 5: THE WARRANT MODEL WITH MARKET CREDIT SPREADS

Both the OPM and the first four alternatives base their calculations on the evolution of Total Equity. In contrast, the Warrant Model bases its calculations on the evolution of the Common share price. To implement this model, we discount the FPs at the risky yields and value the conversion features as warrants. The model back-solves for the value of Common per share to set the per share value of Series C to \$5.00.<sup>11</sup>

$$\$5.0 = \frac{\$5.00}{1.215^5} + C(S, \$5.00, \ln(1.015), 107\%, 5)$$

S is the price per share of Common. The value of S that yields a Series C value of \$5.00 is \$4.12.

$$\frac{\$5.00}{1.215^5} + \$3.11 = \$5.00$$

This model also has an inconsistency. The value of Common, \$4.12, which exceeds that of Series A, \$3.98, is highlighted in yellow. Since the Preferred has priority in payout and is convertible to Common, it should never be worth less than the Common.

## MODEL 5: FIXED PAYMENTS DISCOUNTED BY RISKY YIELDS + WARRANT

	Series C	Series B	Series A	Common	Total Equity
FP	\$5.00	\$2.00	\$1.00		
Spread	20.0%	24.0%	29.0%		
FP value	\$1.89	\$0.64	\$0.26		
Warrant value	\$3.11	\$3.50	\$3.71		
Total per share value	\$5.00	\$4.14	\$3.98	\$4.12	
Number of shares (mm)	3.0	5.0	5.0	5.0	
Security Values	\$15.0	\$20.7	\$19.9	\$20.6	\$76.2

<sup>11</sup> The volatility for Common of 107% is equivalent to a Total Equity volatility of 100% for Total Equity.

## MODEL 6: THE WARRANT MODEL WITH TF DISCOUNTING

In this model, the exercise price is discounted by the risk-free rate when a Warrant is in-the-money. When it is out-of-the-money, its risky yield is used.<sup>12,13</sup> We back-solve for the Common per-share value. Note that the common volatility is estimated at 107% similar to Model 5.

$$\$5.0 = \frac{\$5.00}{1.015^5} N(d_2) + \frac{\$5.00}{1.215^5} (1 - N(d_2)) + C(S, \$5.00, \ln(1.015), 107\%, 5)$$

The value of S that produces a Series C value of \$5.00 is \$3.80.

$$\frac{\$5.00}{1.015^5} 10.0\% + \frac{\$5.00}{1.215^5} 90.0\% + \$2.84 = \$5.00$$

## MODEL 6: FIXED PAYMENTS DISCOUNTED IN THE TF MANNER + WARRANT

	Series C	Series B	Series A	Common	Total Equity
FP	\$5.00	\$2.00	\$1.00		
Spread	20.0%	24.0%	29.0%		
$N(d_2)$	10.0%	18.4%	27.0%		
Risk-free rate PV of FP	\$0.5	\$0.3	\$0.3		
$1 - N(d_2)$	90.0%	81.6%	73.0%		
Risk-yield PV of FP	\$1.70	\$0.52	\$0.19		
Probability-weighted PV of FP	\$2.16	\$0.87	\$0.44		
Warrant value	\$2.84	\$3.21	\$3.41		
Total per share value	\$5.00	\$4.07	\$3.85	\$3.80	
Number of shares (mm)	3.0	5.0	5.0	5.0	
Security values	\$15.0	\$20.4	\$19.3	\$19.0	\$73.6

<sup>12</sup> Note that we are not applying the downside credit spreads. Had we used the downside credit spreads as the basis for risky yields the model would merely produce numerical results similar to those of Model 5.

<sup>13</sup> A variation of this approach could consider applying discounts to the volatility inputs consistent with what is done in practice in valuing the convertible bonds of public companies.

# SUMMARY & CONCLUSIONS

The table below summarizes the valuation results. All six models increase the valuations of Total Equity and Common relative to the traditional OPM. They also shift the credit spreads toward the assumed market rates. In those regards, they are potential alternatives to the OPM. Models 3 and 4 suffer from an internal inconsistency, highlighted in orange. The magnitude of the inconsistency in relative terms, however, appears not to be significant. Model 5 assigns a lower value to Series A than to Common, which is a no-arbitrage violation highlighted in yellow. Appraisers can raise plausible objections to applying the TF approach to discounting FPs in Models 4 and 6. Models 3 through 6 all have both positive and negative features.

Models 1 and 2 represent refinements to the traditional OPM approach by back-solving for volatility rather than relying on empirical data. This change in emphasis deserves serious consideration. Comparable publicly traded companies are not comparable in many ways. To reflect non-comparability, appraisers often make ad hoc adjustments to the empirical data, typically to increase volatility. It may be preferable to consider broader ranges for credit spreads and volatilities based on market data, then select the most reasonable values within those ranges. Reasonableness should be evaluated using factors such as the ratio of the value of common over the senior preferred, the ratio of the value of liquidation preferences over the aggregate value of each preferred class, secondary transactions, and other relevant indicators.

Judgment related to the appropriate method and assumptions must be exercised based on various company-specific factors. The valuation model must be adapted to the specific facts and circumstances of the company and securities being assessed.

## SUMMARY OF VALUATION RESULTS

Model		1	2	3	4	5	6
		One OPM Volatility	Three Volatilities	Usual Discounting + OPM	TF Discounting + OPM	Usual Discounting + Warrant	TF Discounting + Warrant
Total Equity		\$67.5	\$71.7	\$74.6	\$77.2	\$70.4	\$73.6
Total Value of Securities		\$67.5	\$71.7	\$74.6	\$75.8	\$72.3	\$73.6
Per Share Value	C	\$5.00	\$5.00	\$5.00	\$5.00	\$5.00	\$5.00
	B	\$3.60	\$3.86	\$4.13	\$4.19	\$4.07	\$4.07
	A	\$3.46	\$3.75	\$3.93	\$4.01	\$3.80	\$3.85
	Common	\$3.44	\$3.73	\$3.86	\$3.97	\$3.60	\$4.12
Spreads	C	15%	20%	20%	20%	20%	20%
	B	28%	35%	24%	24%	24%	24%
	A	33%	40%	29%	29%	29%	29%
Volatility for	1	100%	114%	115%	100%	100%	
Break	2	100%	114%	108%	100%	100%	107%
Points	3 to 6	100%	114%	106%	100%	100%	

## ABOUT VRC

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